SPECIAL FUNCTIONS AS ANTILOGARITHMS OF SECOND ORDER

Danuta Przeworska-Rolewicz

When computers came along and everyone was talking about the end of mathematics, it was thought that special functions would be one of the first causalities. The opposite happened: special functions are going great guns as never before...

Gian-Carlo Rota,
Indiscrete Thoughts, 1997

Abstract

The purpose of the present paper is to show that the so-called special functions of Mathematical Physics can be obtained by means of antilogarithms of the second order for the usual differential operator $\frac{\mathrm{d}}{\mathrm{d}t}$. The same method applied to a right invertible operator D in a commutative Leibniz algebra with logarithms permits to determine eigenvectors of linear equations of order two in D with coefficients in the algebra X under consideration by a reduction to the generalized Sturm-Liouville operator. It seems that, in a sense, the proposed method is an answer for the question of Gian-Carlo Rota concerning a unified approach to special functions (cf. [R1], Problem 4). Note that, in general, we do not need any assumption about the Hilbert structure of the algebra X.

Mathematics Subject Classification: 33Cxx, 34B30, 47A75, 47E05

Key Words and Phrases: exponential, sine mapping, cosine mapping, logarithmic mapping of higher order, antilogarithmic mapping of higher order, symmetric differential operator of order two, Sturm-Liouville operator, eigenvector, special function, associate and orthogonal special functions, special element, hypergeometric functions, Legendre, Gauss, Chebyshev, Jacobi, Laguerre, Hermite functions, Helmholtz, Neumann, Hankel, Bessel, Euler, Gegenbauer, Weber, Airy, Ince, Mathieu, Sturm-Liouville equations and/or functions